An example of Kaluza-Klein-like theory with boundary conditions, which lead to massless and mass protected spinors chirally coupled to gauge fields

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Abstract

The genuine Kaluza-Klein-like theories (with no fields in addition to gravity) have difficulties with the existence of massless spinors after the compactification of some of dimensions of space[1]. We assume a $M^{(1+3)}\times$ a flat finite disk in (1+5)-dimensional space, with the boundary allowing spinors of only one handedness. Massless spinors then chirally couple to the corresponding background gauge gravitational field, which solves equations of motion for a free field, linear in the Riemann curvature.

Introduction: Genuine Kaluza-Klein-like theories, assuming nothing but a gravitational field in d-dimensional space (no additional gauge or scalar fields), which after the spontaneous compactification of a $(d-4)$-dimensional part of space manifest in four dimensions as all the known gauge fields including gravity, have difficulties[1] with masslessness of fermionic fields at low energies. It looks namely very difficult to avoid after the compactification of a part of space the appearance of representations of both handedness in this part of space and consequently also in the (1+3)-dimensional space. Accordingly, the gauge fields can hardly couple chirally in the (1+3) - dimensional space.

In more popular versions, in which one only uses the idea of extra dimensions but does not use gravity fields themselves to make gauge fields, by just having gauge fields from outset, the break of the parity symmetry in the compactified part of space is achieved, for instance, by (an outset of) magnetic fields[2]. Since gravity does not violate parity, also typically not in extra dimensions alone, it looks accordingly impossible to make the genuine Kaluza-Klein gauge particles coupled chirally[1, 3]. The most popular string theories, on the other side, have such an abundance of "fundamental" (or rather separate string states) gauge fields, that there is (absolutely) no need for the genuine Kaluza-Klein ones.

In an approach by one of us[4, 5] it has long been the wish to obtain the gauge fields from only gravity, so that "everything" would become gravity. This approach has taken the inspiration from looking for unifying all the internal degrees of freedom, that is the spin and all the charges, into only the spin. This approach is also a kind of the genuine Kaluza-Klein theory, suffering the same problems, with the problem of getting chiral fermions included, unless we can solve them.

There are several attempts in the literature, which use boundary conditions to select massless fields of a particular[6, 7, 8, 9]. Boundary conditions are chosen by choosing discrete orbifold symmetries.

In this letter we study a toy model with a Weyl spinor, which caries in $d(=1+5)$ -dimensional space with the symmetry $M^{(1+3)}\times$ a flat finite disk only the spin as the internal degree of freedom. On the boundary of a finite disk only spinors of one handedness are allowed. The only back ground field is the gravitational gauge field with vielbeins and spin connections, which manifest the rotational symmetry on the flat disk. We demonstrate that there exist spinors, which manifest in $M^{(1+3)}$ masslessness (have no partners of opposite handedness) and are chirally coupled by the Kaluza-Klein charge to the corresponding

Kaluza-Klein field. The current through the wall is for all the spinors (the solutions of the Weyl equation in $d(=1+5)$-dimensional space, which manifest in $d(=1+3)$-dimensional space as massless or massive spinors) equal to zero. The Kaluza-Klein charge of all the spinors is proportional to the total angular momentum on the disk.

The assumed background field with the spin connections and vielbeins in $d=(1+5)$ (flat on the disk and preserving the rotational symmetry on the disk up to gauge transformations) fulfills the equations of motion, which follow from the action in $d=(1+5)$, linear in the Riemann curvature. The action manifests the Kaluza-Klein $U(1)$ gauge field term.

Weyl spinors in gravitational fields with spin connections and vielbeins: We let[12][14] a spinor interact with a gravitational field through vielbeins $f^{\alpha}{}_{a}$ (inverted vielbeins to $e^{a}{}_{\alpha}$ with the properties $e^{a}{}_{\alpha} f^{\alpha}{}_{b} = \delta^{a}{}_{b},\ e^{a}{}_{\alpha} f^{\beta}{}_{a} = \delta^{\beta}_{\alpha}$) and spin connections, namely $\omega_{ab\alpha}$, which is the gauge field of $S^{ab} = \frac{i}{4}(\gamma^a \gamma^b - \gamma^b \gamma^a)$. We choose the basic states in the space of spin degrees of freedom to be eigen states of the Cartan sub algebra of the operators: S^{03}, S^{12}, S^{56}.

The covariant momentum of a spinor is taken to be

$$p_{0a} \;=\; f^{\alpha}{}_{a} p_{0\alpha}, \quad p_{0\alpha}\psi = p_{\alpha} - \frac{1}{2} S^{cd} \omega_{cd\alpha}, \tag{1}$$

when applied to a spinor function ψ.

A kind of a total covariant derivative of $e^{a}{}_{\alpha}$ (a vector with both-Einstein and flat index) is taken to be

$$p_{0\alpha} e^{a}{}_{\beta} = i e^{a}{}_{\beta;\alpha} = i(e^{a}{}_{\beta,\alpha} + \omega^{a}{}_{d\alpha} e^{d}{}_{\beta} - \Gamma^{\gamma}{}_{\beta\alpha} e^{a}{}_{\gamma}). \tag{2}$$

We require that this derivative of a vielbein is zero: $e^{a}{}_{\beta;\alpha} = 0$.

The corresponding Lagrange density $\mathcal{L}$ for a Weyl has the form $\mathcal{L} = E\frac{1}{2}[(\psi^{\dagger}\gamma^0\gamma^a p_{0a}\psi) + (\psi^{\dagger}\gamma^0\gamma^a p_{0a}\psi)^{\dagger}]$ and leads to

$$\mathcal{L} \;=\; E\psi^{\dagger}\gamma^0\gamma^a (p_a - \frac{1}{2} S^{cd}\Omega_{cda})\psi, \tag{3}$$

with $E = \det(e^{a}{}_{\alpha})$, $\Omega_{cda} = \frac{1}{2}(\omega_{cda} + (-)^{cda}\omega^{*}{}_{cda})$, and with $(-)^{cda}$, which is -1, if two indices are equal, and is 1 otherwise (if all three indices are different). (In $d=2$ case Ω_{abc} is always pure imaginary.)

The Lagrange density (3) leads to the Weyl equation

$$\gamma^0\gamma^a P_{0a}\psi = 0, \quad P_{0a} = (f^{\alpha}{}_{a} p_{\alpha} - \frac{1}{2} S^{cd}\Omega_{cda}). \tag{4}$$

Taking now into account that $\gamma^a \gamma^b = \eta^{ab} - 2iS^{ab}$, $\{\gamma^a, S^{bc}\}_- = i(\eta^{ab}\gamma^c - \eta^{ac}\gamma^b)$, one easily finds that[15] $\gamma^a P_{0a} \gamma^b P_{0b} = P_{0a} P^a_0 + \frac{1}{2} S^{ab} S^{cd} \mathcal{R}_{abcd} + S^{ab} \mathcal{T}^c{}_{ab} P_{0c}$. We find $\mathcal{R}_{abcd} = f^\alpha{}_{[a} f^\beta{}_{b]} (\Omega_{cd\alpha,\beta} + \Omega_{ce\alpha} \Omega^e{}_{d\beta})$ and for the torsion: $\mathcal{T}^c{}_{ab} = f^\alpha{}_{[a} (f^\beta{}_{b]})_{,\alpha} e^c{}_\beta + \Omega_{[a}{}^c{}_{b]}$.

The most general vielbein for $d = 2$ can be written by an appropriate parameterization as

$$e^s{}_\sigma = e^{\varphi/2} \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}, f^\sigma{}_s = e^{-\varphi/2} \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}, \tag{5}$$

with $s = 5, 6$ and $\sigma = (5), (6)$ and $g_{\sigma\tau} = e^\varphi \eta_{\sigma\tau}, g^{\sigma\tau} = e^{-\varphi} \eta^{\sigma\tau}$, $\eta_{\sigma\tau} = diag(-1, -1) = \eta^{\sigma\tau}$. If there is no dilatation then $E = 1$. If in the case of $d = 2$, the Einstein action for a free gravitational field $S = \int\ d^dx\ E\ R$, with $R = f^{\sigma[s} f^{\tau t]}\ \Omega_{st\sigma,\tau}$ is varied with respect to both, spin connections and vielbeins, the corresponding equations of motion bring no conditions on any of these two types of fields, so that any zweibein and any spin connection can be assumed. Accordingly, if we put the expression in Eq.(2) equal to zero, can this equation *for* $d = 2$ *be understood as the defining equation for* $\mathcal{T}^s{}_{\sigma\tau} \equiv \Gamma^s{}_{\sigma\tau} - \Gamma^s{}_{\tau\sigma}$, for any spin connection Ω_{stu} and any zweibein.

We assume that a two dimensional space is a flat disk

$$f^\sigma{}_s = \delta^\sigma{}_s,\ \omega_{56s} = 0, \tag{6}$$

with the rotational symmetry and with the radius ρ_0. We require that spinors must obey the boundary condition

$$(1 - i n^{(\rho)}{}_\alpha n^{(\varphi)}{}_\beta f^\alpha{}_a f^\beta{}_b \gamma^a \gamma^b) \psi|_{\rho=\rho_0} = 0, \tag{7}$$

where ψ is the solution of the Weyl equation in $d = 1 + 5$ and $n^{(\rho)} = (0, 0, 0, 0, \cos\varphi, \sin\varphi)$, $n^{(\varphi)} = (0, 0, 0, 0, -\sin\varphi, \cos\varphi)$ are the two unit vectors perpendicular and tangential to the boundary (at ρ_0), respectively. We shall see in what follows that the boundary forces massless spinors - that is spinors, which manifest masslessness in $(1 + 3)$-dimensional space - to be of only right handedness[16], while the current at the boundary is in the perpendicular direction equal to zero for either massless or massive spinors. Spinors manifest masslessness in $d = (1 + 3)$-dimensional space, if they solve the Weyl equation (4) with $a = 5, 6$, so that the term $E\psi^\dagger \gamma^0 \gamma^s p_{0s} \psi$, $s = 5, 6$ (the only term, which would manifest as a mass term in $d = 1 + 3$) is equal to zero. The boundary condition assures the mass protection.

Weyl spinors on a flat disk: The Weyl spinor wave functions ψ, manifesting masslessness in the $(1+3)$ space, must on a flat disk obey the Weyl equations of motion (Eq.(4)), while the Weyl wave functions in $d=1+5$, manifesting in $d=1+3$ a mass m, obey the Dirac equation

$$\gamma^5 e^{2i\phi S^{56}}(p_{0(5)}+2iS^{56}p_{0(6)})\psi=-m\psi. \tag{8}$$

For our flat disk $\phi=0$ and $\omega_{st\sigma}=0$. The Dirac equation in $d=2$ goes to the Weyl equation, if we assume $m=0$. The Weyl wave function in $d=1+5$, may (in our case for a (flat disk) $\times M^{1+3}$) be written as a product of a function, which depends only on the coordinates $x^\sigma, \sigma=(5),(6)$, and the part, which we denote by $(+)$ and $[-]$ and includes the dependence on all the internal degrees of freedom (in (1+5)) as well as on the coordinates $x^\mu, \mu=(0),(1),(2),(3)$. It follows that $(+)$ and $[-]$ are the two types of the spinor states, corresponding to the eigen values of S^{56} equal to $+1/2$ and $-1/2$, respectively (what ever the spinor and coordinate content in $d=1+3$ might be). Accordingly $(+)$ represents the right and $[-]$ the left handed spinor state in $d=2$ ($\Gamma^{(2)}=2S^{56}$), while in $d=(1+3)$ they represent the left and the right handed spinors, respectively (since we started in $d=(1+5)$ with the left handed Weyl spinor).

We introduce the polar coordinates in $d=2$: $x^{(5)}=\rho\cos\varphi,\ x^{(6)}=\rho\sin\varphi$ and require that states are also eigen states of the total angular momentum $M^{56}=-i\frac{\partial}{\partial\varphi}+S^{56}$. We then write eigen states of the Dirac equation (8) as

$$\begin{aligned}
\psi^m{}_{+(n+1/2)}(\rho,\varphi) &= \alpha^+{}_n(\rho)e^{in\varphi}(+)+\beta^+{}_{n+1}(\rho)e^{i(n+1)\varphi}[-],\\
\psi^m{}_{-(n+1/2)}(\rho,\varphi) &= \alpha^-{}_{n+1}(\rho)e^{-i(n+1)\varphi}(+)+\beta^-{}_n(\rho)e^{-in\varphi}[-].
\end{aligned} \tag{9}$$

Index m denotes that spinors carry a mass m. The functions $\alpha^\pm{}_k,\ \beta\pm_k,\ k=n,n+1$, $n=0,1,2,3...$, must solve Eq.(8) (for either the massive or the massless case), which in the polar coordinates read

$$\begin{aligned}
i(\frac{\partial}{\partial\rho}-\frac{n}{\rho})\alpha^+{}_n(\rho) &= m\,\beta^+{}_{n+1}(\rho),\\
i(\frac{\partial}{\partial\rho}+\frac{n+1}{\rho})\beta^+{}_{n+1}(\rho) &= m\,\alpha^+{}_n(\rho).
\end{aligned} \tag{10}$$

One immediately finds that for $m=0$ the functions $\alpha^+{}_n(\rho)=\rho^n$, $\beta^+{}_{n+1}=0$ and $\alpha^-{}_{n+1}=0$, $\beta^-{}_n=\rho^n$, $n=0,1,2...$, solve Eq.(10), so that the right handed solutions and the left handed

solutions in $d=2$ are as follows

$$\psi^{m=0}{}_{+(n+1/2)}(\rho,\varphi) = A_{+(n+\frac{1}{2})}\rho^n e^{in\varphi}(+),$$
$$\psi^{m=0}{}_{-(n+1/2)}(\rho,\varphi) = A_{-(n+\frac{1}{2})}\rho^n e^{-in\varphi}[-], \tag{11}$$

with $A_{\pm(n+\frac{1}{2})}$ constants. In the massive case, for the two types of functions $\alpha^{\pm}{}_m$ and $\beta^{\pm}{}_m$ the Bessel functions can be taken as follows: $\alpha^{+}{}_n(\rho) = \beta^{-}{}_n(\rho) = J_n$ and $\beta^{+}{}_{n+1}(\rho) = \alpha^{-}_{n+1}(\rho) = J_{n+1}$. The zeros of the Bessel functions are at $m\rho_0 = 2, 4.., 3.8.., 5.1.., etc$, corresponding to $J_0, J_1, J_2, ...$, respectively.

Boundary conditions: According to the boundary condition of Eq.(7), which in our case indeed requires that

$$0 = (1-\Gamma^{(2)})\psi^{m}{}_{\pm(n+1/2)}|_{\rho=\rho_0}, \tag{12}$$

only the masses, for which $\beta^{\pm}{}_k(\rho_0) = 0$, are allowed, since the term with $(+)$ is at ρ_0 multiplied by zero, while the term with $[-]$ is multiplied by (1+1). In the massless case, the boundary condition requires that $A_{-(n+\frac{1}{2})} = 0$, so that only right handed spinors with the spin part $(+)$ survive. There are accordingly infinite number of massive and of massless solutions. To different solutions different total angular moments correspond and in the massive case also different masses.

We easily see that a current through the wall

$$n^{(\rho)}{}_{\alpha} j^{\alpha}|_{\rho=\rho_0} = \psi^{+}\gamma^0\gamma^a f^{\alpha}{}_a n^{(\rho)}{}_{\alpha}\psi|_{\rho=\rho_0}, \tag{13}$$

is in all the cases (massless and massive) equal to zero. In the massive case, the current is proportional to the terms $\alpha^{\pm}{}_k(\rho_0)\beta^{\pm}{}_{k\pm1}(\rho_0)$, which are zero, since always either $\alpha^{\pm}{}_k(\rho_0)$ or $\beta^{\pm}{}_{k\pm1}(\rho_0)$ is zero on the wall. In the massless case $\beta^{+}{}_m$ is zero all over.

Spinors coupled to gauge fields in $M^{(1+3)} \times M^{(2)}$, with $M^{(2)}$ a flat finite disk: To study how do spinors couple to the Kaluza-Klein gauge fields in the case of $M^{(1+5)}$, "broken" to $M^{(1+3)}\times$ a flat disk with ρ_0 and the boundary condition, which allows only right handed spinors at ρ_0, we first look for (background) gauge gravitational fields, which preserve the rotational symmetry on the disk. To find such fields, we study the coordinate transformations of the type $x'^{\mu} = x^{\mu}, x'^{\sigma} = x^{\sigma} + \zeta^{\sigma}(x^{\tau})\theta(x^{\mu})$, with[17] $\zeta^{\sigma}(x^{\tau}) = \zeta^{\sigma}{}_0 + \varepsilon^{\sigma}{}_{\tau}x^{\tau}$. We start with $f^{\alpha}{}_a = \delta^{\alpha}{}_a, \omega_{ab\alpha} = 0$. Requiring globaly that $\delta_0 f^{s}{}_{\sigma} = 0$ $(\delta_0 f^{\sigma}{}_a = \omega_a{}^b\ f^{\sigma}{}_b + \zeta^{\sigma}{}_{,\tau}\ \theta(x^{\mu})\ f^{\tau}{}_a - \theta(x^{\mu})\ \zeta^{\tau}\ f^{\sigma}{}_{a,\tau})$ and $\delta_0\omega_{st\sigma} = 0$ $(\delta_0\omega_{st\mu} = \omega_s{}^a\ \omega_{at\mu} + \omega_t{}^b\ \omega_{sb\mu} - \zeta^{\tau}{}_{,\mu}\ \theta(x^{\nu})\ \omega_{st\tau} - \theta(x^{\nu})\ \zeta^{\tau}\ \omega_{st\mu,\tau} -$

$\omega_{st,\mu}$) we end up with $\omega_{st,\sigma} = 0, \zeta^{\sigma}{}_{0} = 0$ and, by replacing $\theta_{,\mu}$ with A_μ (which is the gauge $U(1)$ field whose gauge transformation leads to $A_\mu + \theta_{,\mu}$), with $\delta_0 f^{\sigma}{}_{m} = A_\mu \delta^{\mu}{}_{m} \varepsilon^{\sigma}{}_{\tau} x^{\tau}$, $\delta_0 \omega_{st\mu} = -\varepsilon_{st} A_\mu$. Accordingly the following background vielbein field

$$e^{a}{}_{\alpha} = \begin{pmatrix} \delta^{m}{}_{\mu} & e^{m}{}_{\sigma} = 0 \\ e^{s}{}_{\mu} & e^{s}{}_{\sigma} \end{pmatrix}, f^{\alpha}{}_{a} = \begin{pmatrix} \delta^{\mu}{}_{m} & f^{\sigma}{}_{m} \\ 0 = f^{\mu}{}_{s} & f^{\sigma}{}_{s} \end{pmatrix}, \tag{14}$$

with $f^{\sigma}{}_{m} = A_\mu \delta^{\mu}{}_{m} \varepsilon^{\sigma}{}_{\tau} x^{\tau}$ and the spin connection field

$$\omega_{st\mu} = -\varepsilon_{st} A_\mu, \quad \omega_{sm\mu} = -\frac{1}{2} F_{\mu\nu} \delta^{\nu}{}_{m} \varepsilon_{s\sigma} x^{\sigma} \tag{15}$$

are assumed. The term $\omega_{sm\mu} = -\frac{1}{2} F_{\mu\nu} \delta^{\nu}{}_{m} \varepsilon_{s\sigma} x^{\sigma} = -\omega_{ms\mu}$, which is proportional to $F^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}$, can not be derived in the above way, since A^{μ}, if pure gauge, contributes zero to it. But this term and all the others are the solution of the equations of motion, which follow from the action, linear in the Riemann curvature, as we shall see bellow. The $U(1)$ gauge field A_μ depends only on x^{μ}. All the other components of the spin connection fields are zero, since for simplicity we allow no gravity in $(1+3)$ dimensional space.

To determine the current, coupled to the Kaluza-Klein gauge fields A_μ, we analyze the spinor action

$$\begin{aligned} \mathcal{S} \;=\; & \int d^d x E \bar{\psi} \gamma^{a} P_{0a} \psi = \int d^d x \bar{\psi} \gamma^{m} \delta^{\mu}{}_{m} p_\mu \psi + \\ & \int d^d x \bar{\psi} \gamma^{m} (-) S^{sm} \omega_{sm\mu} \psi + \int d^d x \bar{\psi} \gamma^{s} \delta^{\sigma}{}_{s} p_\sigma \psi + \\ & \int d^d x \bar{\psi} \gamma^{m} \delta^{\mu}{}_{m} A_\mu (\varepsilon^{\sigma}{}_{\tau} p_\sigma + S^{56}) \psi. \end{aligned} \tag{16}$$

ψ are defined in $d = (1+5)$ dimensional space (as solutions of the Weyl equation) and solve the Dirac (massive - if $\bar{\psi} \gamma^{s} \delta^{\sigma}{}_{s} p_\sigma \psi = -m$) or the Weyl (massless - if $\bar{\psi} \gamma^{s} \delta^{\sigma}{}_{s} p_\sigma \psi = 0$) equation in $d = 2$ (the terms $(+)$ and $[-]$ determine the spin part in $d = 1+5$ and also the dependence on x^{μ}). E is for $f^{\alpha}{}_{a}$ from (14) equal to 1. The first term on the right hand side of Eq.(16) is the kinetic term (together with the last term defines the covariant derivative $p_{0\mu}$ in $d = 1+3$). The second term on the right hand side contributes nothing when integration over the disk is performed, since it is proportional to x^{σ} ($\omega_{sm\mu} = -\frac{1}{2} F_{\mu\nu} \delta^{\nu}{}_{m} \varepsilon_{s\sigma} x^{\sigma}$).

We end up with

$$j^{\mu} = \int d^2 x \bar{\psi} \gamma^{m} \delta^{\mu}{}_{m} M^{56} \psi \tag{17}$$

as the current in $d = 1+3$. *The charge in $d = 1+3$ is obviously proportional to the total angular momentum $M^{56} = L^{56} + S^{56}$ on a disk*, for either massless or massive spinors. One

notices, that our toy model allows massless spinors of any angular momentum in $d=2$, which then means that spinors of charges, proportional to $n+1/2$, for any n are allowed.

Gauge fields on $M^{(1+3)}\times$ a finite disk: One can check that the gauge field in Eqs.(14,15) is in agreement with the relation between $\omega_{ab\alpha}$ and $f^{\alpha}{}_{a}$, which follow when varrying the action for a free gauge field, if linear in the Riemann curvature ($\int d^dxER$), with respect to the spin connection field $\omega_{ab\alpha}$: $\omega_{ab\alpha} = -\frac{1}{2E}\Big\{e_{e\alpha}e_{b\gamma}\,\partial_\beta(Ef^{\gamma[e}f^{\beta}{}_{a]}) + e_{e\alpha}e_{a\gamma}\,\partial_\beta(Ef^{\gamma}{}_{[b}f^{\beta e]}) - e_{e\alpha}e^{e}{}_{\gamma}\,\partial_\beta(Ef^{\gamma}{}_{[a}f^{\beta}{}_{b]})\Big\} - \frac{1}{d-2}\Big\{e_{a\alpha}\frac{1}{E}e^{d}{}_{\gamma}\partial_\beta\left(Ef^{\gamma}{}_{[d}f^{\beta}{}_{b]}\right) - e_{b\alpha}\frac{1}{E}e^{d}{}_{\gamma}\partial_\beta\left(Ef^{\gamma}{}_{[d}f^{\beta}{}_{a]}\right)\Big\}$. For the particular solution of a general $(1+5)$ case, when $f^{\mu}{}_{m}=\delta^{\mu}{}_{m}$ and $\omega_{mn\mu}=0$, which concerns the case with no gravity in $(1+3)$ space, the Riemann tensor simplifies to $R=-\frac{1}{2}F^{\mu\nu}F_{\mu\nu}x^{\sigma}x_{\sigma}$, which after the integration over $\rho\; d\rho d\varphi$ leads to the known action for the gauge $U(1)$ field. One can check that the boundary term contributes zero.

Conclusions: We start with one Weyl spinor of only one handedness in a space M^{1+5}, and assume that the space factorizes into $M^{(1+3)}\times$ a flat finite disk with the radius ρ_0 and with the boundary, which allows only spinors of a particular handedness: $(1 - in^{(\rho)}{}_{\alpha}n^{(\varphi)}{}_{\beta}f^{\alpha}{}_{a}f^{(\beta)}{}_{b}\gamma^{a}\gamma^{b})\psi|_{\rho=\rho_0} = 0$. The spinor, whose only internal degree of freedom is the spin, interacts with the gauge gravitational field represented by spin connections ($\omega_{ab\alpha}$) and vielbeins ($f^{\alpha}{}_{a}$). The disk (manifesting the rotational symmetry) is flat ($f^{\sigma}{}_{s}=\delta^{\sigma}{}_{s}$, $\omega_{st\sigma}=0$). We look for massless spinors in $(1+3)$ "physical" space, which are mass protected and chirally coupled to a Kaluza-Klein gauge field through a quantized (proportional to an integer) Kaluza-Klein charge.

To be massless in $(1+3)$ space, spinors must obey the Weyl equation on a disk: $\gamma^0\gamma^s f^{\sigma}{}_{s}p_{\sigma}\psi = 0, s=\{5,6\}, \sigma=\{(5),(6)\}$. The boundary condition on the disk makes the current of (massless and massive) spinors in the perpendicular direction to be zero and guarantees that massless spinors are mass protected.

The background gauge field, chosen to obey isometry relations and respecting accordingly the rotational symmetry on the disk, fullfils the general equations of motion, which follow in $(1+5)$ from the action, linear in the Riemann curvature. The effective Lagrangean in $d=(1+3)$ is for the flat space the ordinary Largangean for the $U(1)$ field. The current (for massless or massive spinors) is in the $(1+3)$-dimensional space proportional to the total angular momentum on the disk (M^{56}), which is accordingly determining the charge of spinors (proportional to $n+1/2, n=0,1,2,..$). Consequently massless spinors are mass protected and chirally coupled to the Kaluza-Klein gauge fields.

The "real" case, with a spin in $d-$dimensional space, which would manifest in (1+3)-dimensional "physical" space the spin and all the known charges, needs, of course, much more than two additional dimensions. All the relations are then much more complex. But some properties will very likely repeat, like: Spinors with only a spin in $(1+(d-1))$-dimensional space will manifest in $(1+(q-1))$-dimensional space the masslessness, together with the mass protection and the charge, if a kind of boundary conditions in a compactified $(d-q)-$dimensional space would made possible the existence of spinors of only one handedness.

It would be worthwhile to find out how our boundary conditions are related (if at all) to the boundary conditions, which introduce orbifolds[6, 7, 8, 9].

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[14] Latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, .., \mu, \nu, ..\sigma, \tau..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indices from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). We assume the signature $\eta^{ab} = diag\{1, -1, -1, \cdots, -1\}$.

[15] $[a...b]$ means, that the expression must be anti symmetrized with respect to a, b.

[16] If we start with the left handed spinor in $d = 1 + 5$, the handedness of a massless spinor in $d = 1 + 3$ is chosen to be the left one.

[17] $\varepsilon^{(5)}{}_{(6)} = -1 = -\varepsilon^{(6)}{}_{(5)}$, while the rest of terms are zero.